# Calculation of alcohol content

If the fermentation reaction is rewritten taking water into account, one has:

x sugar + (1-x) water → 2xy alcohol + 2xy $CO_2$ + x(1-y) sugar + (1-x) water

where y represents the proportion of sugar turned into alcohol, called attenuation.

| | Molar mass M (g/mol) | Density d | (d-1)/d | Molar volume (L/mol) (= M/d) |
|---|---|---|---|---|
| Sugar | 180 | 1,55 | 0,355 | 0,116 |
| Water | 18 | 1 | 0 | 0,018 |
| Alcohol | 46 | 0,79 | -0,2595 | 0,058 |
| Honey | | 1,41 | 0,2908 | |
| $CO_2$ (gas) | 44 | 0,0018 | -554,6 | 24 |

*Table 8: some properties of sugar, alcohol, etc.*

One can notice that $\frac{M_s}{2M_a} \approx 1{,}96$ and $\frac{d_s}{d_a} \approx 1{,}95$.

So $\frac{d_s}{d_a}\frac{2M_a}{M_s} \approx 1$, that is the volume

$$2xyV_a + x(1-y)V_s + (1-x)V_w = xV_s + (1-x)V_w + x\frac{M_s}{d_s}\left[\frac{d_s}{d_a}\frac{2M_a}{M_s} - 1\right]y$$

is constant.

By definition the initial density (specific gravity) is the initial mass of the must (water +sugar) divided by the initial volume: $d_i = \frac{V_w}{M_w}\frac{xM_s + (1-x)M_w}{xV_s + (1-x)V_w}$. Which can be rewritten:

$$d_i = 1 + \frac{\frac{M_s}{M_w}\frac{d_s - 1}{d_s}x}{1 + \left(\frac{M_s}{M_w}\frac{1}{d_s} - 1\right)x}$$

It is also possible to get x as a function of the initial density: $\frac{1}{x} - 1 = \frac{M_s}{M_w}\frac{1}{d_s}\frac{d_s - d_i}{d_i - 1}$

Final density is by definition:

$$d_f = \frac{V_w}{M_w}\frac{2xyM_a + x(1-y)M_s + (1-x)M_w}{2xyV_a + x(1-y)V_s + (1-x)V_w}$$

This can be simplified using that the volume is constant:

$$d_f \approx 1 + \left[1 - y\frac{d_s - d_a}{d_s - 1}\right](d_i - 1)$$

This expression can be modified to get the attenuation as a function of $d_i$ and $d_f$:

$$y \approx \frac{d_s - 1}{d_s - d_a}\frac{d_i - d_f}{d_i - 1} \approx 0{,}73\frac{d_i - d_f}{d_i - 1}$$

The alcohol by volume is equal to the volume of alcohol (in ml) per 100 ml mead:

$$ABV = \frac{2xyV_a}{2xyV_a + x(1-y)V_s + (1-x)V_w}$$

$$\approx \frac{d_i - d_f}{d_s - d_a} \approx 1{,}32\,(d_i - d_f)$$

So the attenuation and the alcohol by volume can be calculated, knowning initial and final densities,.

The sugar by volume is:

$$SBV = \frac{x(1-y)V_s}{x(1-y)V_s + 2xyV_a + (1-x)V_w}$$

$$\approx (1-y)\frac{d_i - 1}{d_s - 1} \approx \frac{d_i - 1}{d_s - 1} - \frac{d_i - d_f}{d_s - d_a}$$

- If all the sugar has been consumed (y = 1) :

$$d_f = 1 - \frac{1 - d_a}{d_s - 1}(d_i - 1) \approx 1 - 0{,}37\,(d_i - 1)$$

$$TV_a \approx \frac{d_i - 1}{d_s - 1}y \approx 1{,}8\,(d_i - 1)y$$

- If d = 1, $y \approx \frac{d_s - 1}{d_s - d_a} \approx 73\%$. So when the density is 1, three quarters or so of the sugar have been turned into alcohol (whatever the initial density is). White labs publishes attenuations for their yeasts of 70-80 %.
- Examples (final density $d_f$ is equal to 0.995 in all examples).

| $d_i$ | y | TVa | TVs | TMs |
|---|---|---|---|---|
| 1,052 | 80,0 % | 7,6 % | 1,9 % | 30 g/L |
| 1,092 | 77,0 % | 12,9 % | 3,9 % | 60 g/L |
| 1,132 | 75,8 % | 18,3 % | 5,8 % | 90 g/L |

*Table 9: some properties of mead as a function of original gravity (final gravity is .995).*

One can notice that there is a lot of sugar remaining but the sugar content does not depend on the final density only. So the final density should not be used, as some people do, as a criterion to determine whether the mead is dry or sweet.

Is there really so much sugar at the end? If so, why do bacteria not attack all meads, not only the sweeter ones?